Related Rates and Maximum and Minimum Values

Math 130 - Essentials of Calculus

4 November 2019

Warm-Up

Example

Assume that x and y are functions of t. If $y = x^3 + 2x$ *and* $\frac{dx}{dt} = 5$, *find* $\frac{dy}{dt}$ *when* $x = 2$.

More Than One Dynamic Quantity

Example

Two cars start moving from the same point. One travels south at $60 mi/hr$ *and the other travels west at* $25 mi/hr$. *At what rate is the distance between the cars increasing two hours later?*

Multiple Relations

Gravel is being dumped from a conveyor belt at a rate of $30 ft^3/min$, and its coarseness is such that it forms a pile in the shape of a cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is $10 ft$ high?

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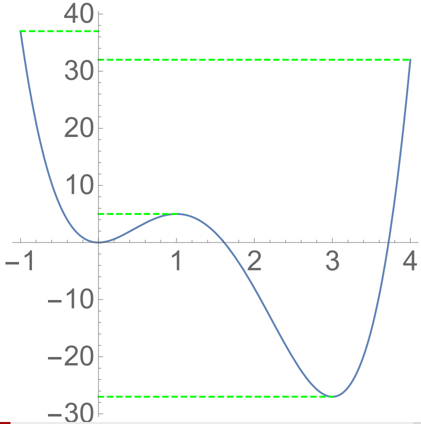
Extrema of a Function

Consider the function $f(x) = 3x^4 - 16x^3 + 18x^2$ on the domain $-1 \leq x \leq 4$. Where are the absolute maximum and absolute minimum values, and what are they? Are there any local minimum and local maximum values?



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It turns out that if you consider a continuous function on a closed interval, of the form $[a, b]$, you're guaranteed an absolute maximum and minimum.

Theorem (The Extreme Value Theorem)

If f *is continuous on a closed interval, then it always attains an absolute maximum value and an absolute minimum value on that interval.*

Locating Extreme Values

Observing some of the pictures we've had so far, the following theorem is apparent:

Theorem (Fermat's Theorem)

If f has a local maximum or minimum at c, and if $f'(c)$ exists, then $f'(c) = 0$.

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It's possible that a function could have a local extrema at a place where $f'(c) \neq 0$, for example, consider $f(x) = |x|$.It turns out that what we're really looking for are *critical numbers*.

Definition (Critical Number)

A critical number of a function f is a number c in the domain of f such that either $f'(c) = 0$ or $f'(c)$ does not exist.